KEK-TH-1600
OIQP-13-01
HUPD-1211

On the Cancellation Mechanism of Radiation from the Unruh detector

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Abstract

A uniformly accelerated detector (Unruh detector) in the Minkowski vacuum is excited as if it is exposed to the thermal bath with temperature proportional to its acceleration. In the inertial frame, since both of an excitation and a deexcitation of the detector are accompanied by emission of radiation into the Minkowski vacuum, one may suspect that the Unruh detector emits radiation like the Larmor radiation from an accelerated charged particle. However, it is known that the radiation is miraculously cancelled by a quantum interference effect. In this paper, we investigate under what condition the radiation cancels out. We first show that the cancellation occurs if the Green function satisfies a relation similar to the Kubo-Martin-Schwinger (KMS) condition. We then study two examples, Unruh detectors in the 3+1 dimensional Minkowski spacetime and in the de Sitter spacetime. In both cases, the relation holds only in a restricted region of the spacetime, but the radiation is cancelled in the whole spacetime. Hence the KMS-like relation is necessary but not sufficient for the cancellation to occur.

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1 Introduction

A uniformly accelerated observer sees the Minkowski vacuum as thermally excited, which is known as the Unruh effect [1]. The Unruh effect is fundamental and important because it is related by the equivalence principle to the thermal behavior of gravity in spacetime with horizons [2]. The Unruh temperature $T_U = \hbar a/2\pi c k_B$ is proportional to its acceleration a and very small for ordinary experimental settings. But the recent developments of ultra-high intense lasers make the Unruh effect experimentally accessible [3]. For example, Chen and Tajima [4] proposed an indirect detection of the Unruh effect by measuring an excess of radiation from an accelerated electron in the electromagnetic field of ultra-high intense lasers. Since the trajectory of a charged particle in acceleration fluctuates around the classical trajectory due to the Unruh effect, it may emit an extra radiation (Unruh radiation) besides the classical Larmor radiation. The idea was further investigated in Ref. [5], in which it is shown that the motion of an electron in uniform acceleration is thermalized and fluctuates around the classical trajectory. An interference effect plays an important role to cancel the Unruh radiation at least partially, though it is not yet established whether the radiation is totally canceled.

A similar analysis was investigated for a uniformly accelerated Unruh detector, i.e. a system of a harmonic oscillator coupled with the quantum field. The backreaction of emission of particles to the detector's trajectory is neglected, so the trajectory of the detector is not a dynamical variable. In [6, 7, 8, 9, 10], it was shown that cancellation of radiation occurs due to an interference effect. Here we briefly explain the mechanism of cancellation. The system is described by quantum scalar field ϕ and the Unruh detector in uniform acceleration. The equation of motion of the scalar field is given by

$$\partial^\mu \partial_\mu \phi(x) = j(x) \tag{1.1}$$

where the scalar current $j(x)$ is induced by the Unruh detector. Its solution $\phi(x)$ is written as a sum of an inhomogeneous and a homogeneous solutions

$$\phi(x) = \phi_{inh}(x) + \phi_h(x). \tag{1.2}$$

The inhomogeneous solution $\phi_{inh}(x)$ describes the field induced by the coupling to the Unruh detector while the homogeneous part $\phi_h(x)$ corresponds to the vacuum fluctuation. The normal-ordered two-point function is given by the sum

$$\langle \phi(x)\phi(y)\rangle - \langle \phi_h(x)\phi_h(y)\rangle = \langle \phi_{inh}(x)\phi_{inh}(y)\rangle + \langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle. \tag{1.3}$$

The first term $\langle \phi_{inh}(x)\phi_{inh}(y)\rangle$ is a classical contribution in the presence of the Unruh detector. In addition to it, we also have the interference terms $\langle \phi_{inh}(x)\phi_h(y)\rangle +$

$\langle\phi_h(x)\phi_{inh}(y)\rangle$. It is purely quantum mechanical and appears as a result of the interference between the induced field ϕ_{inh} and the vacuum fluctuation ϕ_h. Since the inhomogeneous solution has its origin in the vacuum fluctuation, the interference terms do not vanish. It is indeed shown that these two contributions are miraculously cancelled each other (apart from the polarization cloud near the detector) in some specific examples.

Although the cancellation is straightforwardly shown, it is not clear under what condition the cancellation generally occurs. An intuitive interpretation of the cancellation is based on an observation that the Unruh detector eventually reaches equilibrium in the thermal bath with the Unruh temperature. This may explain why the total energy flux vanishes. It sounds plausible, but it cannot answer to the following two questions. First, the uniformly accelerated observer can only see a part of the spacetime, the right Rindler wedge (see also section 3). So it is not certain whether the cancellation occurs also in the future wedge to which the accelerated observer is inaccessible. Second, such classical interpretation of the cancellation cannot explain why the cancellation occurs for an inertial observer. In the thermal equilibrium, the energy balance is reached by processes of absorption and emission. But, for an inertial observer, only emission can occur since there are no particles to be absorbed in the vacuum. Both absorption and emission processes for the uniformly accelerated observer correspond to emission of a quanta for the inertial observer. Hence, the cancellation of the energy flux appears mysterious.

In this paper, we first investigate the mechanism of the cancellation and find a condition for the cancellation. The condition is similar to the Kubo-Martin-Schwinger (KMS) relation in a thermal system. This makes clear and explicit the relation between the thermal properties of the uniformly accelerated observer and the cancellation of radiation. We then consider two examples, the Unruh detector in 3+1 dimensional Minkowski spacetime and a detector moving along a geodesic in de Sitter spacetime. In both cases we show that a two-point function has different behaviors in different wedges across the Rindler horizon of the detector. The above condition for the cancellation is only satisfied in a restricted region of the spacetime (the right Rindler wedge). This may reflect the fact that the uniformly accelerated observer can only access to the restricted part of the spacetime. In the future wedge, we show that, although the above thermal condition is not satisfied, radiation also cancels out and only a polarization cloud remains.

The paper is organized as follows. In Section 2, we give a general condition for the cancellation of radiation to occur. In Section 3, we explicitly demonstrate how the condition works to cancel the radiation from an Unruh detector moving at a constant acceleration in the Minkowski spacetime. We also consider a detector at rest in the de Sitter spacetime.

Section 4 is devoted to conclusions and discussions.

2 KMS-like condition

We consider a coupled system of a scalar field $\phi(x)$ and a harmonic oscillator whose action is given by

$$\begin{aligned} S[Q,\phi;z] &= \frac{m}{2}\int d\tau \left((\dot{Q}(\tau))^2 - \Omega_0^2 Q^2\right) + \frac{1}{2}\int d^n x\sqrt{|g|}\left(\partial^\mu\phi(x)\partial_\mu\phi(x) + F(R)\phi^2\right) \\ &\quad +\lambda\int d^n x d\tau\ P[Q(\tau)]\phi(x)\delta^n(x - z(\tau)). \end{aligned} \tag{2.1}$$

$Q(\tau)$ is a harmonic oscillator with a mass m and an angular frequency Ω_0, and denotes the dynamical degree of freedom of the Unruh detector. Its world line trajectory is given by $x^\mu = z^\mu(\tau)$. Note that $z^\mu(\tau)$ is not a dynamical variable. $\phi(x)$ is coupled to the Unruh detector though the last term. $F(R)$ is a function of the Riemann scalar curvature. $P[Q]$ is defined by

$$P[Q(\tau)] = \sum_j p_j \left(\frac{d}{d\tau}\right)^j Q(\tau), \tag{2.2}$$

where p_i is a constant.

The Heisenberg equations of motion are given by

$$m\left(\ddot{Q}(\tau) + \Omega_0^2 Q(\tau)\right) = \lambda\bar{P}[\phi(z(\tau))], \tag{2.3}$$

$$\left(\nabla^\mu\nabla_\mu\phi(x) - F(R)\right)\phi(x) = \frac{\lambda}{\sqrt{|g|}}\int d\tau' P[Q(\tau')]\delta^n(x - z(\tau')), \tag{2.4}$$

where $\bar{P}[Q(\tau)] = \sum_j p_j \left(-\frac{d}{d\tau}\right)^j Q(\tau)$ is a conjugate of P. An inhomogeneous solution of (2.4) is given by

$$\phi_{inh}(x) = \lambda\int d\tau' P[Q(\tau')]G_R(x, z(\tau')), \tag{2.5}$$

where the retarded Green function $G_R(x,y)$ satisfies

$$\left(\nabla^\mu\nabla_\mu - F(R)\right)G_R(x,y) = \frac{\delta^n(x-y)}{\sqrt{|g|}} \tag{2.6}$$

with an appropriate boundary condition. A general solution is then written as a sum of an inhomogeneous and a homogeneous solutions

$$\phi(x) = \phi_{inh}(x) + \phi_h(x). \tag{2.7}$$

Since we are considering an open system[§], the homogeneous solution $\phi_h(x)$ describes the vacuum fluctuation and satisfies

$$\left(\nabla^\mu \nabla_\mu - F(R)\right) \phi_h(x) = 0. \tag{2.8}$$

Substituting the solution (2.7) into the equation of motion for $Q(\tau)$, Eq. (2.3), we obtain the following equation,

$$m\left(\frac{d^2}{d\tau^2} + \Omega_0^2\right) Q(\tau) - \lambda^2 \bar{P}\left[\int d\tau' P[Q(\tau')] G_R(z(\tau), z(\tau'))\right] = \lambda \bar{P}[\phi_h(z(\tau))]. \tag{2.9}$$

It describes a stochastic behavior of the Unruh detector, and the detector eventually reaches the thermal equilibrium at the Unruh temperature. The second term in the l.h.s. gives a dissipation due to the emission of radiation while the r.h.s. gives a stochastic noise of the quantum vacuum fluctuation. In the following, we will consider a trajectory of the detector so that $G_R(z(\tau), z(\tau'))$ is a function of $\tau - \tau'$,

$$G_R(z(\tau), z(\tau')) = G_R(\tau - \tau').$$

This condition holds in two examples studied in section 3. After the Unruh detector reaches the thermal equilibrium, the coupled system becomes stationary (but not necessarily static). We then Fourier-transform $Q(\tau)$ as

$$Q(\tau) \;=\; \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega\tau} \tilde{Q}(\omega). \tag{2.10}$$

Similarly the vacuum fluctuation $\phi_h(z(\tau))$ along the trajectory of the detector can be Fourier transformed as

$$\tilde{\phi}(\omega) = \int d\tau \; e^{i\omega\tau} \phi_h(z(\tau)). \tag{2.11}$$

Then the equation (2.9) can be solved in terms of $\tilde{\phi}(\omega)$ as

$$\tilde{Q}(\omega) = \lambda h(\omega) \tilde{\phi}(\omega). \tag{2.12}$$

Here $h(\omega)$ is given by

$$h(\omega) = \frac{f(-\omega)}{-m\omega^2 + m\Omega_0^2 - \lambda^2 f(\omega) f(-\omega) \tilde{G}_R(\omega)}, \tag{2.13}$$

where we defined $f(\omega) = \sum_j p_j (-i\omega)^j$, and

$$\tilde{G}_R(\omega) = \int d(\tau - \tau') G_R(\tau - \tau') e^{i\omega(\tau - \tau')} = \tilde{G}_R^*(-\omega). \tag{2.14}$$

[§]If we consider a closed system (such as a system confined in a small box), we need to take into account a nonequilibrium evolution of the ϕ field by using the in-in formalism. Then the homogeneous solution deviates from the vacuum fluctuation.

Note that the relation $h(-\omega) = h^*(\omega)$ holds.

Now we consider the renormalized two-point function Eq. (1.3). The energy-momentum flux can be obtained from the two-point function by differentiating it with respect to x and y. Hence we focus our investigation on the two-point function.

Using (2.12), the inhomogeneous solution ϕ_{inh} (2.5) can be written in terms of the homogeneous one (vacuum fluctuation) $\tilde{\phi}(\omega)$ as

$$\phi_{inh}(x) = \lambda^2 \int d\tau_x \int \frac{d\omega_x}{2\pi} f(\omega_x) h(\omega_x) \tilde{\phi}(\omega_x)\; G_R(x, z(\tau_x)) e^{-i\omega_x \tau_x}. \tag{2.15}$$

Then the two-point correlation of the inhomogeneous solution is given by

$$\begin{aligned} \langle \phi_{inh}(x)\phi_{inh}(y)\rangle &= \lambda^4 \int \frac{d\tau_x d\omega_x d\tau_y d\omega_y}{(2\pi)^2}\; G_R(x, z(\tau_x)) G_R(y, z(\tau_y)) e^{-i(\omega_x\tau_x + \omega_y\tau_y)} \\ &\qquad \times\, h(\omega_x) f(\omega_x) h(\omega_y) f(\omega_y) \langle \tilde{\phi}(\omega_x)\tilde{\phi}(\omega_y)\rangle. \end{aligned} \tag{2.16}$$

Since the inhomogeneous solution ϕ_{inh} is written in terms of the homogeneous one ϕ_h, the interference between them gives a nonvanishing contribution to the two-point function. It is given by

$$\begin{aligned} & \langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle \\ = \;& \lambda \int d\tau_x G_R(x, z(\tau_x)) \langle P[Q(\tau_x)]\phi_h(y)\rangle + \lambda \int d\tau_y G_R(y, z(\tau_y)) \langle \phi_h(x) P[Q(\tau_y)]\rangle \\ = \;& \lambda^2 \int \frac{d\tau_x d\omega_x}{2\pi} G_R(x, z(\tau_x)) e^{-i\omega_x\tau_x} h(\omega_x) f(\omega_x) \langle \tilde{\phi}(\omega_x)\phi_h(y)\rangle \\ & + \lambda^2 \int \frac{d\tau_y d\omega_y}{2\pi} G_R(y, z(\tau_y)) e^{-i\omega_y\tau_y} h(\omega_y) f(\omega_y) \langle \phi_h(x)\tilde{\phi}(\omega_y)\rangle. \end{aligned} \tag{2.17}$$

Comparing (2.16) and (2.17), one can see that (2.16) is written in terms of the correlation of vacuum fluctuations on the trajectory $\langle \phi_h(z(\tau))\phi_h(z(\tau'))\rangle$ while (2.17) depends on $\langle \phi_h(x)\phi_h(z(\tau))\rangle$, and a nontrivial relation is necessary to make them related.

In the remaining of this section, we show that the following relation plays an important role to cancel out the radiation. The key relation we will use is

$$\langle \tilde{\phi}(\omega)\phi_h(y)\rangle = \rho(\omega)\langle [\tilde{\phi}(\omega), \phi_h(y)]\rangle, \tag{2.18}$$

where $\rho(\omega)$ is a real function of ω. Or equivalently,

$$\langle \tilde{\phi}(\omega)\phi_h(y)\rangle = \alpha(\omega)\langle \phi_h(y)\tilde{\phi}(\omega)\rangle, \quad \alpha(\omega) = \frac{\rho(\omega)}{\rho(\omega) - 1}. \tag{2.19}$$

These relations show that $\langle \tilde{\phi}(\omega)\phi_h(y)\rangle$ and $\langle [\tilde{\phi}(\omega), \phi_h(y)]\rangle$ have the same y-dependence up to a real function of ω. Both of them satisfy the homogeneous equation $(\nabla^\mu\nabla_\mu - F(R))\, G(y) = 0$, but it does not mean that the relation (2.18) is always satisfied.

We now prove that the radiation in the inhomogeneous term (2.16) and the interference term (2.17) cancels out when the key relation Eq. (2.18) holds. Introducing a function $G(x,y)$ by

$$G(x,y) = -i\langle[\phi_h(x),\phi_h(y)]\rangle = -G_R(x,y) + G_A(x,y), \tag{2.20}$$

(see e.g., [11]), we find that

$$\begin{aligned}
\langle\tilde{\phi}(\omega_x)\tilde{\phi}(\omega_y)\rangle &= \int d\tau_y \ \langle\tilde{\phi}(\omega_x)\phi_h(z(\tau_y))\rangle e^{i\omega_y\tau_y} \\
&= i\int d\tau_x d\tau_y \ \rho(\omega_x)G(\tau_x-\tau_y)e^{i(\omega_x\tau_x+\omega_y\tau_y)} \\
&= 4\pi\delta(\omega_x+\omega_y)\rho(\omega_x) \ \mathrm{Im}\tilde{G}_R(\omega_x).
\end{aligned} \tag{2.21}$$

In the last equality we used the relation $G_A(x,y) = G_R(y,x)$ and $\tilde{G}(\omega) = -\tilde{G}_R(\omega) + \tilde{G}_A(\omega) = -2i\,\mathrm{Im}\tilde{G}_R(\omega)$, where $\tilde{G}(\omega)$ and $\tilde{G}_A(\omega)$ are the Fourier transforms of $G(z(\tau),z(\tau')) = G(\tau-\tau')$ and $G_A(\tau(z),\tau(z')) = G_A(\tau-\tau')$, respectively. Substituting this relation into Eq. (2.16), we have

$$\begin{aligned}
&\langle\phi_{inh}(x)\phi_{inh}(y)\rangle \\
&= \lambda^4\int\frac{d\tau_x d\tau_y d\omega}{2\pi}G_R(x,z(\tau_x))G_R(y,z(\tau_y))|h(\omega)f(\omega)|^2\rho(\omega)e^{-i\omega(\tau_x-\tau_y)}2\ \mathrm{Im}\tilde{G}_R(\omega).
\end{aligned} \tag{2.22}$$

On the other hand, by using the key relation (2.18) the interference term (2.17) becomes

$$\begin{aligned}
&\langle\phi_{inh}(x)\phi_h(y)\rangle + \langle\phi_h(x)\phi_{inh}(y)\rangle \\
&= -i\lambda^2\int\frac{d\tau_x d\tau_y d\omega}{2\pi}e^{-i\omega(\tau_x-\tau_y)}\rho(\omega) \\
&\quad\times\left(G_R(x,z(\tau_x))G(y,z(\tau_y))f(\omega)h(\omega) - G(x,z(\tau_x))G_R(y,z(\tau_y))f(-\omega)h(-\omega)\right),
\end{aligned} \tag{2.23}$$

where we used

$$\langle\phi_h(x)\tilde{\phi}(\omega)\rangle = (\langle\phi_h(x)\tilde{\phi}(\omega)\rangle^*)^* = (\langle\tilde{\phi}(-\omega)\phi_h(x)\rangle)^* = \rho(-\omega)\int d\tau(-i)G(z(\tau),x)e^{i\omega\tau}. \tag{2.24}$$

Since the commutator $G(x,y)$ is written as $G(x,y) = G_A(x,y) - G_R(x,y)$, (2.23) can be decomposed into a term containing a product of two G_R and the other with a product of G_R and G_A. It can be easily shown by using the identity

$$\begin{aligned}
h(\omega)f(\omega) - h(-\omega)f(-\omega) &= \lambda^2|h(\omega)|^2\left(|f(\omega)|^2\tilde{G}_R(\omega) - |f(\omega)|^2\tilde{G}_R(-\omega)\right) \\
&= |h(\omega)f(\omega)|^2 2i\lambda^2\ \mathrm{Im}\tilde{G}_R(\omega)
\end{aligned} \tag{2.25}$$

that the term containing a product of two G_R in (2.23) totally cancels the two-point function $\langle\phi_{inh}(x)\phi_{inh}(y)\rangle$ in (2.22), which would generate the classical radia1tion by the fluctuating motion of the accelerated detector.

As a result of the above cancellation, the renormalized two-point function becomes

$$\begin{aligned}
& \langle\phi(x)\phi(y) - \phi_h(x)\phi_h(y)\rangle \\
= \; & -i\lambda^2 \int \frac{d\tau_x d\tau_y d\omega}{2\pi} e^{-i\omega(\tau_x-\tau_y)}\rho(\omega) \\
& \times \{G_R(x, z(\tau_x))G_A(y, z(\tau_y))f(\omega)h(\omega) - G_A(x, z(\tau_x))G_R(y, z(\tau_y))f(-\omega)h(-\omega)\}\,.
\end{aligned} \tag{2.26}$$

It contains a product of G_R and G_A, and because of this we can show that the energy-momentum tensor derived from this two-point function damps faster than the behavior of radiation. Hence it does not give an energy flux at infinity. We will see this explicitly in the next section.

3 Two Examples

In this section we consider two examples to investigate the mechanism of cancellation of radiation. We will see that the key relation (2.18) holds only in a restricted region of the spacetime. The first example is the Unruh detector that is uniformly accelerated in the 3+1 dimensional Minkowski spacetime. The second one is a detector fixed at the origin of the spatial coordinates in the 3+1 de Sitter spacetime.

3.1 Unruh detector in Minkowski spacetime

We consider a massless scalar field (2.1) in the Minkowski spacetime . For simplicity, we take $P[Q(\tau)] = Q(\tau)$ so that $f(\omega) = 1$. The vacuum two-point function $\langle\phi(x)\phi(y)\rangle$ is a function of the invariant distance $\sigma = (x - y)^2$. The trajectory of a uniformly accelerated detector is given by

$$z^\mu(\tau) = \left(\frac{\sinh a\tau}{a}, \frac{\cosh a\tau}{a}, 0, 0\right), \tag{3.1}$$

and the invariant distance between two points, $z^\mu(\tau)$ and $z^\mu(\tau')$, on the trajectory is given by

$$\sigma = (z(\tau) - z(\tau'))^2 = \frac{4}{a^2}\left(\sinh\frac{a(\tau-\tau')}{2}\right)^2. \tag{3.2}$$

It is a function of the difference of the detector's proper time, $\tau - \tau'$. Therefore, the Green function is a function of $(\tau - \tau')$; $G_R(z(\tau), z(\tau')) = G_R(\tau - \tau')$.

In 3+1 dimensional Minkowski spacetime, the Wightman function is given by

$$\langle\phi_h(x)\phi_h(y)\rangle = -\frac{1}{4\pi^2}\frac{1}{(x-y)^2 - i\epsilon(x^0-y^0)} \tag{3.3}$$

where ϵ is an infinitesimally small positive constant. To explore when the key relation (2.18) holds, we calculate the following quantity,

$$\begin{aligned}\langle\phi_h(x)\tilde{\phi}(\omega)\rangle &= \int d\tau e^{i\omega\tau}\langle\phi_h(x)\phi_h(z(\tau))\rangle \\ &= -\frac{1}{4\pi^2}\int d\tau\frac{e^{i\omega\tau}}{(x^0-z^0(\tau)-i\epsilon)^2-(x^1-z^1(\tau))^2-(x^2)^2-(x^3)^2}. \end{aligned}\tag{3.4}$$

The integrand has poles on the complex τ plane, whose positions are obtained by solving the equation

$$\left(x^0 - \frac{\sinh(a\tau)}{a}\right)^2 - \left(x^1 - \frac{\cosh(a\tau)}{a}\right)^2 - (x^2)^2 - (x^3)^2 = 0. \tag{3.5}$$

In terms of the lightcone coordinates $u = x^0 - x^1$ and $v = x^0 + x^1$, it becomes

$$-u\frac{e^{a\tau}}{a} + v\frac{e^{-a\tau}}{a} + x^\mu x_\mu - \frac{1}{a^2} = 0. \tag{3.6}$$

The solutions of (3.6) are obtained in a different form depending on a given spacetime point. We consider two types of observers, one in the future wedge where $u>0$ and $v>0$ and the other in the right Rindler wedge where $u<0$ and $v>0$.

If x is in the right Rindler wedge with $u<0$ and $v>0$, there are two types of solutions and each of them satisfies the following equation,

$$e^{a\tau_-^R} = \frac{a}{2|u|}\left(L^2 - \sqrt{L^4 - \tfrac{4}{a^2}|uv|}\right) > 0 \tag{3.7}$$

$$e^{a\tau_+^R} = \frac{a}{2|u|}\left(L^2 + \sqrt{L^4 - \tfrac{4}{a^2}|uv|}\right) > 0, \tag{3.8}$$

where $L^2 = -x^\mu x_\mu + 1/a^2$. Due to the thermal property of the accelerated observer, the solutions are periodically located at $\tau_\pm^R = \zeta_\pm^R + 2\pi ni/a$, where $\zeta_\pm^R$ are real-valued and defined by

$$\zeta_\pm^R(x) = \frac{1}{a}\ln\left[\frac{a}{2|u|}\left(L^2 \pm \sqrt{L^4 - \tfrac{4}{a^2}|uv|}\right)\right], \tag{3.9}$$

respectively. On the other hand, if x^μ is in the future wedge with $u>0$ and $v>0$, the solutions satisfy

$$e^{a\tau_-^F} = \frac{a}{2u}\left(-L^2 + \sqrt{L^4 + \tfrac{4}{a^2}uv}\right) > 0 \tag{3.10}$$

$$e^{a\tau_+^F} = \frac{a}{2u}\left(-L^2 - \sqrt{L^4 + \tfrac{4}{a^2}uv}\right) < 0. \tag{3.11}$$

The solutions are located periodically on the complex τ plane as $\tau_-^F = \zeta_-^F + 2\pi ni/a$ and $\tau_+^F = \zeta_+^F + \pi(2n+1)i/a$, where real-valued $\zeta_\pm^F$ are defined by

$$\zeta_\pm^F(x) = \frac{1}{a}\ln\left[\frac{a}{2|u|}\left(\pm L^2 + \sqrt{L^4 - \tfrac{4}{a^2}|uv|}\right)\right], \tag{3.12}$$

respectively. Note that the imaginary parts of τ_+^F are different by π/a from the other poles. Since the imaginary parts of τ_+^F are half integers divided by a, it is not a proper time of the detector's trajectory. Rather one can interpret it as the proper time of a virtual trajectory in the left Rindler wedge. (For further details, see Figure 2 in [5] .)

Summing these contributions to the integration, we obtain

$$\langle\phi_h(x)\tilde{\phi}(\omega)\rangle = \frac{i}{4\pi l(x)}\frac{1}{e^{2\pi\omega/a}-1}\left(e^{i\omega\zeta_-(x)} - e^{i\omega\zeta_+(x)}Z(\omega,x)\right), \tag{3.13}$$

where

$$Z_x = e^{\pi\omega/a}\theta(u) + \theta(-u), \tag{3.14}$$

$$l(x) = \dot{z}(\zeta_-)\cdot(x - z(\zeta_-)) = \sqrt{\tfrac{a^2}{4}L^4 + uv}, \tag{3.15}$$

Since the two-point function has different behaviors in the right Rindler wedge ($u < 0$) and in the future wedge ($u < 0$), we treat the them separately in the following.

3.1.1 Right Rindler wedge

In the right Rindler wedge with $u < 0$ and $v > 0$, it is easy to verify that (2.18) does hold

$$\langle\tilde{\phi}(\omega)\phi_h(x)\rangle = (\langle\phi_h(x)\tilde{\phi}(-\omega)\rangle)^* = e^{2\pi\omega/a}\langle\phi_h(x)\tilde{\phi}(\omega)\rangle. \tag{3.16}$$

Hence $\rho(\omega) = 1/(1 - e^{-2\pi\omega/a})$. The right Rindler wedge is a region accessible by the accelerated observer. For the accelerated observer, the Minkowski vacuum is seen as a thermal bath which makes the Unruh detector in thermal equilibrium. Therefore the key relation and accordingly the cancellation of radiation is physically understandable in terms of the thermal behavior of the Unruh detector.

Let us now show that the remaining term in the two-point function (2.26) damps faster than the behavior expected for radiation so that it describes a polarization cloud around the detector. The integral over τ_x and τ_y in (2.26) can be performed by using the following identities,

$$\int d\tau G_R(x - z(\tau))q(\tau) = \frac{1}{4\pi l(x)}q(\tau_-),$$

$$\int d\tau G_A(x - z(\tau))q(\tau) = \frac{1}{4\pi l(x)}q(\tau_+), \tag{3.17}$$

where $q(\tau)$ is an arbitrary function and $l(x)$ defined in (3.15) is the distance measured by the comoving observer between x and $z(\tau_-)$. Then, (2.26) becomes

$$\begin{aligned}
&\langle \phi(x)\phi(y) - \phi_h(x)\phi_h(y) \rangle \\
= \quad & -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{\rho(\omega)}{(4\pi)^2 l(x)l(y)} \left\{ e^{-i\omega(\zeta_-^R(x)-\zeta_+^R(y))} h(\omega) - e^{-i\omega(\zeta_+^R(x)-\zeta_-^R(y))} h(-\omega) \right\}.
\end{aligned} \tag{3.18}$$

The integral over ω can be evaluated by summing the residues of the poles of the functions $h(\omega)$ and $\rho(\omega)$. The function $h(\omega)$ is given in (2.13), setting $f(\omega) = 1$. Since the retarded Green function in 3+1 dimensions is given by

$$G_R(x) = \frac{\theta(x^0)\delta(x^\mu x_\mu)}{2\pi},$$

$\tilde{G}_R(\omega)$ becomes

$$\tilde{G}_R(\omega) = \int d\tau e^{i\omega\tau} \frac{\delta((z(\tau) - z(\tau'))^2)}{2\pi} = \int d\tau e^{i\omega\tau} \frac{\delta(\tau - \tau')}{4\pi|\tau - \tau'|}. \tag{3.19}$$

The divergence in the real part of $\tilde{G}_R(\omega)$ gives a renormalization of Ω_0. We write the renormalized frequency as Ω. The imaginary part is given by $\omega/4\pi$, and we have

$$h(\omega) = \frac{1}{-m\omega^2 + m\Omega^2 - i\frac{\omega\lambda^2}{4\pi}}. \tag{3.20}$$

The positions of the poles of $h(\omega)$ are hence given by

$$\omega_\pm = -\frac{i\lambda^2}{8\pi m} \pm \sqrt{\Omega^2 - \frac{\lambda^4}{64m^2\pi^2}}. \tag{3.21}$$

Since both of the poles $\omega_\pm$ are located on the lower complex plane of ω, their contributions to the integral (2.26) become proportional to $\theta(\zeta_-(x) - \zeta_+(y))$ or $\theta(\zeta_-(y) - \zeta_+(x))$, which vanish when two points x, y coincide. Hence they do not give any contributions to the energy momentum tensor. The poles of $\rho(\omega) = 1/(1-e^{-2\pi\omega/a})$ are located at $\omega_{\pm n} = \pm nai$ with a positive integer n. The pole at $\omega = 0$ doesn't give any contribution because the residue vanishes. The pole at $\omega_{\pm n}$ ($\omega \neq 0$) gives a term proportional to $e^{-an|\zeta_-(x)-\zeta_+(y)|}$ or $e^{-an|\zeta_-(y)-\zeta_+(x)|}$ and it damps quickly at infinity. Indeed, we have

$$\begin{aligned}
e^{-a|\zeta_-^R - \zeta_+^R|} \quad &= \quad -\frac{a^2}{4uv} \left(L^2 - \sqrt{L^4 + \frac{4}{a^2}uv} \right)^2 \\
&= \quad -\frac{a^2}{4uv} \left(\frac{2}{a} l(x) - \frac{2}{a}\sqrt{l(x)^2 - uv} \right)^2 \longrightarrow -\frac{uv}{l(x)^2},
\end{aligned} \tag{3.22}$$

which damps faster than $l(x)^{-1}$ at the infinity $l(x) \to \infty$. Together with $l(x)l(y)$ in the denominator of (3.18), the two-point function damps faster than radiation which should behave as $\sim l(x)^{-2}$. Hence there is no radiation in the right Rindler wedge.

3.1.2 Future wedge

In the future wedge with $u < 0$ and $v > 0$, we have

$$\begin{aligned}\langle\tilde{\phi}(\omega)\phi_h(x)\rangle &= (\langle\phi_h(x)\tilde{\phi}(-\omega)\rangle)^* = \frac{i}{4\pi l(x)}\frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a}-1}\left(e^{i\omega\zeta_-^F} - e^{i\omega\zeta_+^F - 2\pi\omega/a}\right) \\ &= e^{2\pi\omega/a}\langle\phi_h(x)\tilde{\phi}(\omega)\rangle + \frac{i}{4\pi l(x)}e^{i\omega\zeta_+^F + \pi\omega/a} \end{aligned} \tag{3.23}$$

and

$$\langle\tilde{\phi}(\omega)\phi_h(x)\rangle = \frac{1}{1-e^{-2\pi\omega/a}}\langle[\tilde{\phi}(\omega), \phi_h(x)]\rangle - \frac{i}{4\pi l(x)}\frac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1}e^{i\omega\zeta_+^F(x)}. \tag{3.24}$$

The key relation (2.18) does not hold exactly because of an additional term involving ζ_+^F. Nevertheless we will show that there is no radiation in the future wedge. The first term in the r.h.s of Eq. (3.24) gives the same contribution to the two-point function (2.26) as discussed in the right Rindler wedge, but since $G_A(x - z(\tau)) = 0$ for x in the future wedge, the contribution vanishes in this case. So only the additional term in (3.24) gives a nonvanishing contribution to the two-point function and we have

$$\begin{aligned}&\langle\phi(x)\phi(y) - \phi_h(x)\phi_h(y)\rangle \\ &= \frac{i\lambda^2}{(4\pi)^2 l(x)l(y)}\int d\omega \frac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1}\left\{h(\omega)e^{-i\omega(\zeta_-^F(x)-\zeta_+^F(y))} - h(-\omega)e^{-i\omega(\zeta_+^F(x)-\zeta_-^F(y))}\right\}.\end{aligned} \tag{3.25}$$

Although the key relation (2.18) does not hold, the final result (3.25) has a similar form to (3.18). In the future wedge, we have the relation

$$\begin{aligned} e^{-a|\zeta_-^F - \zeta_+^F|} &= \frac{a^2}{4uv}\left(L^2 - \sqrt{L^4 + \frac{4}{a^2}uv}\right)^2 \\ &= \frac{a^2}{4uv}\left(\frac{2}{a}l(x) - \frac{2}{a}\sqrt{l(x)^2 - uv}\right)^2 \longrightarrow \frac{uv}{l(x)^2}. \end{aligned} \tag{3.26}$$

Hence the two-point function damps as $l(x)^{-3}$ at the infinity $l(x) \to \infty$, and there is no radiation in the future wedge either.

In the future wedge, the key relation which reflects the thermal behavior of the Unruh detector does not hold, but the cancellation of the radiation still holds. It is interesting that the additional term in (3.24) gives a contribution to the two-point function which is similar to the thermal contribution in the right Rindler wedge. The remaining term describes a polarization cloud induced by the presence of the accelerated Unruh detector.

3.2 Detector at rest in de Sitter spacetime

De Sitter spacetime is the maximally symmetric curved spacetime and the quantum field theory in de Sitter spacetime is the key to understand the early evolution of the universe. It is known that the quantum field theory in the de Sitter spacetime exhibits a similar feature as the Rindler noise. Namely, a detector at rest in de Sitter spacetime sees the Bunch-Davies vacuum as a thermally excited state with the Gibbons-Hawking temperature. In this subsection, we explicitly show that the radiation from the detector cancels out due to the interference effect and that the same theoretical structure reappears as in the Unruh detector in the Minkowski spacetime.

The de Sitter spacetime with a flat spatial chart is given by the line element,

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2, \tag{3.27}$$

where $a(t) = e^{Ht}$ is the scale factor and H is a constant. We consider a detector defined in Eq. (2.1), where it is fixed at the origin of the spatial coordinate, and its trajectory is defined by

$$z^\mu(\tau) = (\tau, 0, 0, 0). \tag{3.28}$$

Hence the proper-time of the detector is the same as the coordinate time t. We consider a real scalar field with the conformal coupling to the curvature and set $F(R) = -R/6$ in Eq. (2.1).

Similar to the case of the Unruh detector in the Minkowski spacetime, we start from evaluating the correlator $\langle\phi_h(x)\tilde{\phi}(\omega)\rangle$. To this end, it is useful to introduce the conformal time η by $\eta = -e^{-Ht}/H = -1/Ha(\eta)$, defined in the range $-\infty < \eta < 0$. The line element is rewritten in a conformally flat form

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2). \tag{3.29}$$

This spatially flat coordinates cover only half the whole de Sitter spacetime (The upper left region in Figure 1).

By defining the variable scaled by the scale factor $\chi(x) = \phi(x)a(x)$, we find that the action for $\phi(x)$ is rephrased to the similar form to that in the Minkowski spacetime

$$S[\phi] = \frac{1}{2}\int d^4x\partial^\mu\chi(x)\partial_\mu\chi(x) \tag{3.30}$$

with the use of the conformal coordinate and $d^4x = d\eta d^3\mathbf{x}$. Then, the two-point function of $\phi_h(x)$ is given by

$$\langle\phi_h(x)\phi_h(y)\rangle = \frac{\langle\chi_h(x)\chi_h(y)\rangle}{a(\eta_x)a(\eta_y)} = -\frac{1}{4\pi^2 a(\eta_x)a(\eta_y)}\frac{1}{(\eta_x-\eta_y)^2 - |\mathbf{x}-\mathbf{y}|^2 - i\epsilon(\eta_x-\eta_y)}, \tag{3.31}$$

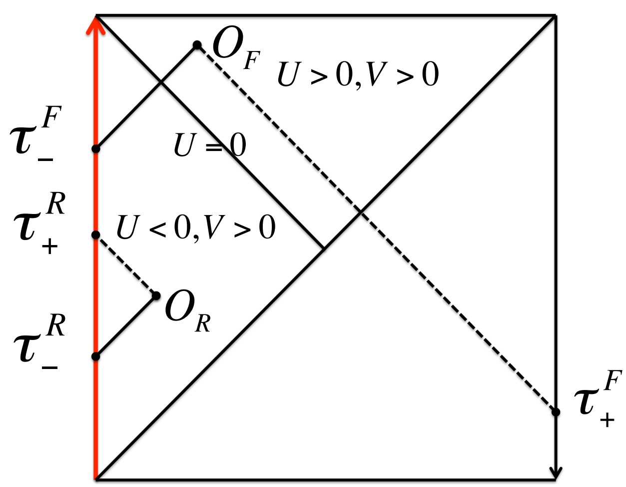

Figure 1: A sketch of the conformal diagram of the de Sitter spacetime. The upper left region is covered by the coordinates of (3.29) with $-\infty < \eta < 0$, while the lower right region is covered by similar coordinates with $0 < \eta < \infty$ (e.g., [12]). In this diagram, the trajectory of the detector is located at the (red) left vertical axis. The null surface $U = 0$ divides the upper left region into the region with $U < 0$ and $V > 0$ (corresponding to the right Rindler wedge) and the region with $U > 0$ and $V > 0$ (corresponding to the future wedge). The points denoted by τ_-^R and τ_+^R correspond to the positions of the solutions (3.35). Similarly, τ_-^F and τ_+^F show the position of the solution of (3.36), but because of the minus sign of the r.h.s. of (3.34) in the region $U > 0$ and $V > 0$, the position of τ_+^F lies on a virtual trajectory lying in the extended spacetime region with a positive conformal time $\eta > 0$.

and $\langle\phi_h(x)\tilde{\phi}(\omega)\rangle$ becomes

$$\begin{aligned}\langle\phi_h(x)\tilde{\phi}(\omega)\rangle &= \int d\tau e^{i\omega\tau}\langle\phi_h(x)\phi_h(z(\tau))\rangle \\ &= \frac{\eta_x H}{4\pi^2}\int d\tau \frac{e^{i\omega\tau - H\tau}}{(\eta_x + e^{-H\tau}/H - i\epsilon)^2 - r_x^2}, \end{aligned}\tag{3.32}$$

with $r_x^2 = |\mathbf{x}|^2$. The poles of the integrand is obtained by solving the equation

$$(\eta_x + e^{-H\tau}/H - i\epsilon)^2 - r_x^2 = 0, \tag{3.33}$$

which yields

$$e^{-H\tau} = H(-\eta_x \pm r_x). \tag{3.34}$$

The structure of the poles on the complex plane of τ depends on a given spacetime point. The solutions are $\tau_\pm^R = \xi_\pm^R(x) + 2\pi ni/H - i\epsilon$ with $n = 0, \pm1, \pm2, \cdots$, for the region $U < 0$ and $V > 0$, which corresponds to the right Rindler wedge, where we defined $r_x + \eta_x = U$, $r_x - \eta_x = V$, and

$$\xi_\pm^R(x) = -\frac{1}{H}\ln\left[H(-\eta_x \mp r_x)\right]. \tag{3.35}$$

On the other hand, the solution are $\tau_-^F = \xi_-^F(x) + 2\pi ni/H - i\epsilon$ and $\tau_+^F = \xi_+^F(x) + 2\pi(n + 1/2)i/H$ with $n = 0, \pm1, \pm2, \cdots$, for the region $U > 0$ and $V > 0$, corresponding to the future wedge, where we defined

$$\xi_\pm^F(x) = -\frac{1}{H}\ln\left[H(\pm\eta_x + r_x)\right]. \tag{3.36}$$

Summing the contributions to the integral, straightforward computations lead to

$$\langle\phi_h(x)\tilde{\phi}(\omega)\rangle = \frac{i}{4\pi a(\eta_x) r_x}\frac{1}{e^{2\pi\omega/H} - 1}\left\{e^{i\omega\xi_-(x)} - e^{i\omega\xi_+^F(x)}e^{\pi\omega/H}\theta(U) - e^{i\omega\xi_+^R(x)}\theta(-U)\right\}, \tag{3.37}$$

which completely agrees with the expression Eq.(3.13) by replacing $a(\eta_x)r_x$ with $l(x)$, and an acceleration constant a with H. Similarly, we also have

$$\langle\tilde{\phi}(\omega)\phi_h(x)\rangle = \frac{i}{4\pi a(\eta_x) r_x}\frac{e^{2\pi\omega/H}}{e^{2\pi\omega/H} - 1}\left\{e^{i\omega\xi_-(x)} - e^{i\omega\xi_+^F(x)}e^{-\pi\omega/H}\theta(U) - e^{i\omega\xi_+^R(x)}\theta(-U)\right\}. \tag{3.38}$$

Thus, the two-point function has a similar structure to the Minkowski case. Namely, the key relation Eq (2.18) is satisfied in the region $U < 0$ and $V > 0$, but an extra term appears in the region $U > 0$ and $V > 0$. Radiation is cancelled in both regions, and the remaining terms damp rapidly at large distance, $a(\eta_x)r_x \to \infty$, as will be demonstrated below. Therefore, the remaining terms in the energy-momentum tensor are regarded as a polarization cloud.

3.2.1 $U < 0$ and $V > 0$

The region corresponds to the right Rindler wedge in the Minkowski spacetime. In the region $U < 0$ and $V > 0$, the key relation Eq. (2.18) is satisfied with $\rho(\omega) = 1/(1 - e^{-2\pi\omega/H})$. In our model in the de Sitter spacetime, we have

$$G_R(x, y) = \frac{1}{a(\eta_x)a(\eta_y)} \frac{\delta(\eta_x - \eta_y - |\mathbf{x} - \mathbf{y}|)}{4\pi|\mathbf{x} - \mathbf{y}|} \tag{3.39}$$

and

$$\begin{aligned}
&\langle \phi_h(x)\phi_{inh}(y) + \phi_{inh}(x)\phi_h(y)\rangle \\
&= \lambda^2 \int d\tau \int \frac{d\omega}{2\pi} e^{-i\omega\tau} h(\omega) \Big(G_R(y, z(\tau))\langle \phi_h(x)\tilde{\phi}(\omega)\rangle + G_R(x, z(\tau))\langle \tilde{\phi}(\omega)\phi_h(y)\rangle \Big)
\end{aligned} \tag{3.40}$$

from Eq. (2.17). Substituting Eqs. (3.38) and (3.39) into (3.40), and using

$$\frac{i}{4\pi a(\eta_x) r_x} e^{i\omega\tau_-(x)} = i \int d\tau' G_R(x, z(\tau')) e^{i\omega\tau'},$$

we find that the inhomogeneous term cancels and that the remaining terms in the renormalized two-point function are given by

$$\begin{aligned}
\langle \phi(x)\phi(y) - \phi_h(x)\phi_h(y)\rangle = &- \frac{i\lambda^2}{(4\pi)^2 a(\eta_x) r_x a(\eta_y) r_y} \\
&\times \int \frac{d\omega}{2\pi} \frac{e^{2\pi\omega/H}}{e^{2\pi\omega/H} - 1} \left\{ h(\omega) e^{-i\omega(\xi_-^R(x) - \xi_+^R(y))} - h(-\omega) e^{-i\omega(\xi_+^R(x) - \xi_-^R(y))} \right\}.
\end{aligned} \tag{3.41}$$

From (3.39), the retarded Green function with the two points on the detector-trajectory is written as

$$G_R(z(\tau), z(\tau')) = \frac{\delta(\tau - \tau')}{4\pi|\tau - \tau'|},$$

and we have (see also [13]),

$$\mathrm{Im}\tilde{G}_R(\omega) = \frac{\omega}{4\pi}. \tag{3.42}$$

Then, the expression of $h(\omega)$ and its pole are the same as those in the Minkowski spacetime with the Unruh detector. Then the poles of $h(\omega)$ in Eq. (3.41) give terms proportional to $\theta(\xi_-^R(x) - \xi_+^R(y))$ or $\theta(\xi_-^R(y) - \xi_+^R(x))$, which vanish in the coincidence limit. The poles from $e^{2\pi\omega/H} - 1 = 0$, $\omega = inH$ with an integer n give a term decreasing rapidly at large distance, and hence do not produce radiation.

3.2.2 $U > 0$ and $V > 0$

The region corresponds to the future wedge in the Minkowski spacetime. In this case, the remaining terms in the renormalized two-point function are given by

$$\langle \phi(x)\phi(y) - \phi_h(x)\phi_h(y)\rangle = -\frac{i\lambda^2}{(4\pi)^2 a(\eta_x) r_x a(\eta_y) r_y} \times \int \frac{d\omega}{2\pi} \frac{e^{\pi\omega/H}}{e^{2\pi\omega/H}-1} \left\{ h(\omega) e^{-i\omega(\xi_-^F(y)-\xi_+^F(x))} - h(-\omega) e^{-i\omega(\xi_+^F(y)-\xi_-^F(x))} \right\}. \tag{3.43}$$

Similar to the above case with $U < 0$, the poles of $h(\omega)$ yield terms proportional to $\theta(\xi_-^F(y) - \xi_+^F(x))$ or $\theta(\xi_-^F(x) - \xi_+^F(y))$, which become zero in the coincidence limit. The poles from $e^{2\pi\omega/H} - 1 = 0$ give terms decreasing rapidly at large distance. Therefore there is no radiation in this region, too.

The surface $U = 0$, which divides the spatially flat de Sitter spacetime into the two regions with $U < 0$ and $U > 0$, corresponds to the cosmological horizon of the detector. Namely, $U = 0$ is equivalent to the relation $r_x a(t_x) = 1/H$, and the detector can never be influenced by the events outside the cosmological horizon classically. In the case of an accelerated detector in the Minkowski spacetime, the detector cannot be influenced by the events in the future wedge beyond the Rindler horizon. This is an analogy between the models in the de Sitter model and in the Minkowski spacetime. In the model of the de Sitter spacetime, however, the radiation emitted from the detector always propagates from the region $U < 0$ to the region $U > 0$ across the surface $U = 0$. Namely any radiation exits the cosmological horizon of the de Sitter spacetime as the universe expands. This may naturally explain the fact that the radiation in the region $U > 0$ vanishes when the radiation in the region $U < 0$ does.

4 Conclusions and Discussions

In this paper, we investigated the mechanism of cancellation of radiation from an Unruh detector coupled with a scalar field. We found that Eq. (2.18) plays a key roll in the cancellation of radiation. The correlation of the inhomogeneous solution is indeed cancelled by an interference effect between the inhomogeneous solution and the vacuum fluctuation when Eq. (2.18) holds. Since the same relation can be derived from the KMS relation in a thermal field theory (see Appendix), the cancellation mechanism of the radiation has its origin in the thermal nature of the uniformly accelerated observer. However, the mechanism of the cancellation of radiation in the future wedge is a bit different. This

may be related to the fact that the uniformly accelerated detector can observe only events in the right Rindler wedge. The key relation Eq. (2.18) does not hold there, and there is an additional contribution to the two-point function. In spite of such differences, the radiation cancels out also in the future wedge. We confirmed these behaviors explicitly in two examples, the Unruh detector in Minkowski spacetime and a detector at rest in de Sitter spacetime.

The cancellation of radiation can be generalized to interacting massive theories at least perturbatively. In presence of interactions, we can calculate the two-point function perturbatively by using Wick contractions. Since we showed rapid damping of the two-point function in a free massless theory, each contribution to the two-point function in perturbative expansions damps more rapidly than the free case so that they cannot give radiation at infinity.

The cancellation of radiation in the right Rindler wedge can be naturally interpreted as a thermal behavior of the detector, in accordance with the fact that the key relation Eq. (2.18) holds there. Indeed it is related to the fact that the Green function is periodic in the imaginary direction of the detector's proper time. (In the Appendix, we see a connection of the key relation to the KMS relation in an ordinary thermal system.) However, the cancellation of radiation in the future wedge is not straightforward. It reminds us of the classical example of the radiation from a uniformly accelerated charged particle. We know that if a particle is uniformly accelerated, it emits Larmor radiation. But, in the accelerated observer's frame, the particle sits at rest but in a constant gravitational field. In this frame, there is no radiation but for the polarization cloud around the charged particle. These two pictures seem to be contradictory to the equivalence principle. The resolution to this paradox is given in [14]. It was shown that the radiation exists only in the future wedge and there is no radiation at all in the right Rindler wedge that the comoving observer can access. This classical example indicates that the cancellation of radiation in the future wedge does not always follow the thermal behavior of the accelerated detector. In the present case, since we are considering a quantum system, the interference effect plays an important role. It will be interesting to further investigate how the detector and the ground state of the quantum field is entangled across the horizon [8].

Acknowledgments

We would like to thank W.G. Unruh, H. Kodama, M. Sasaki, T. Tanaka, J. Garriga and S. Ichinose for useful conversation. The research by S.I. is supported in part by Grant-in-Aid for Scientific Research (19540316) from MEXT, Japan. We are also supported

in part by "the Center for the Promotion of Integrated Sciences (CPIS) " of Sokendai. K.Y. acknowledges useful discussions during the YITP Long-term workshop YITP-T-12-03 on "Gravity and Cosmology 2012", and the support by Grant-in-Aid for Scientific researcher of Japanese Ministry of Education, Culture, Sports, Science and Technology (No. 21540270 and No. 21244033).

A On Equation (2.18) and The KMS Relation

In this appendix, we show that the same relation as Eq. (2.18) is derived from the KMS relation in the thermal field theory. The KMS relation in the thermal field theory is written as

$$G^{\pm}_{\beta}(t, \overrightarrow{x}) = G^{\mp}_{\beta}(t \pm i\beta, \overrightarrow{x}), \tag{1.1}$$

where we denote $G^{+}_{\beta}(x, y) = \langle \phi(x)\phi(y)\rangle_{\beta}$ and $G^{-}_{\beta}(x, y) = \langle \phi(y)\phi(x)\rangle_{\beta}$, and $\langle \cdot \rangle_{\beta}$ denotes the thermal average in the thermal state with the temperature $T = 1/\beta$. One can derive the KMS relation as follows,

$$\begin{aligned}
\langle \phi(t, \overrightarrow{x})\phi(t', \overrightarrow{x}')\rangle_{\beta} &= tr[e^{-\beta H}\phi(t, \overrightarrow{x})\phi(t', \overrightarrow{x}')]/tr[e^{-\beta H}] \\
&= tr[e^{-\beta H}\phi(t, \overrightarrow{x})e^{\beta H}e^{-\beta H}\phi(t', \overrightarrow{x}')]/tr[e^{-\beta H}] \\
&= tr[e^{-\beta H}\phi(t', \overrightarrow{x}')\phi(t + i\beta, \overrightarrow{x})]/tr[e^{-\beta H}] \\
&= \langle \phi(t', \overrightarrow{x}')\phi(t + i\beta, \overrightarrow{x})\rangle_{\beta}.
\end{aligned} \tag{1.2}$$

From this relation, it is easy to show that

$$\langle \tilde{\phi}(\omega)\phi(x)\rangle_{\beta} = e^{\beta\omega}\langle \phi(x)\tilde{\phi}(\omega)\rangle_{\beta}, \tag{1.3}$$

which is equivalent to

$$\langle \tilde{\phi}(\omega)\phi(x)\rangle_{\beta} = \frac{1}{1 - e^{-\beta\omega}}\langle [\tilde{\phi}(\omega), \phi(x)]\rangle_{\beta}. \tag{1.4}$$

In this derivation, the periodicity in the direction of the imaginary time is important.

In both examples in section 3 of the present paper, the Green function is periodic in the imaginary direction of the detector's proper time, since, in the Unruh detector in Minkowski spacetime, the trajectory is written in term of $e^{\pm a\tau}$, and in the de Sitter space, the conformal time is written as $\eta = -e^{-Ht}/H$. These periodic behaviors are similar to the above thermal property, but contrary to the KMS relation, the ordering of the fields are not interchanged. So the periodicity itself does not lead to the KMS-like relation (2.18).

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